Math Virtual Learning

Calculus AB

Volume: The Disk Method

April 22, 2020

Calculus AB

Lesson: April 22, 2020

Objective/Learning Target:

Students will calculate the volume of a solid of revolution using the disk method.

Warm-Up:

Watch Videos: **Disk method around x-axis**
Disk method around y-axis

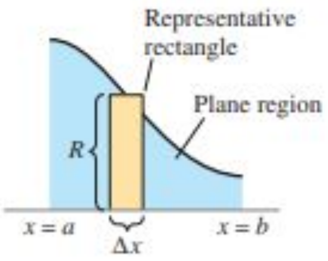
Read Article: Disk Method

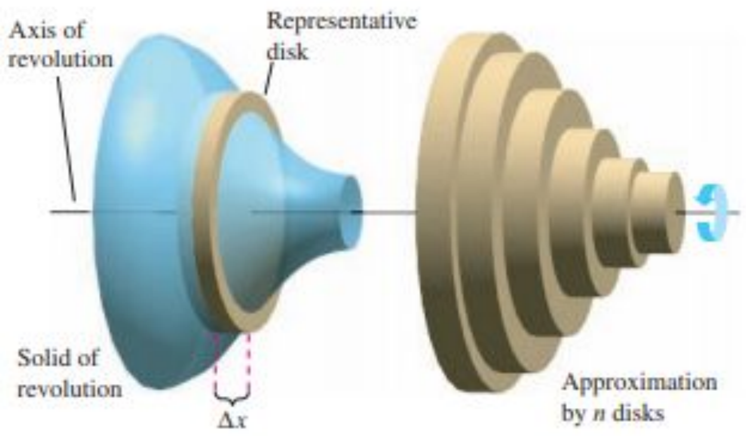
Notes:

The Disk Method

The volume of the solid formed by revolving the region bounded by the graph of f and the x-axis $(a \le x \le b)$ about the x-axis is

$$\text{Volume} = \pi\int_a^b [f(x)]^2\,dx.$$





Approximation by n rectangles

Examples:

EXAMPLE 1 **Finding the Volume of a Solid of Revolution**

Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = -x^2 + x$ and the x-axis about the x-axis.

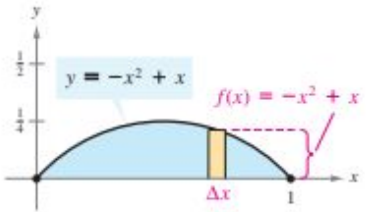
SOLUTION Begin by sketching the region bounded by the graph of f and the x-axis. As shown in Figure 5.27(a), sketch a representative rectangle whose height is $f(x)$ and whose width is Δx. From this rectangle, you can see that the radius of the solid is

$$\text{Radius} = f(x) = -x^2 + x.$$

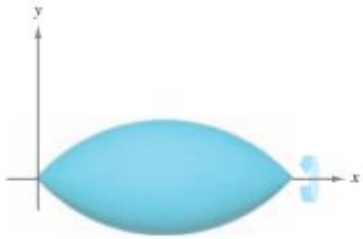
Using the Disk Method, you can find the volume of the solid of revolution.

$$\begin{aligned}
\text{Volume} &= \pi\int_0^1 [f(x)]^2\,dx && \text{Disk Method} \\
&= \pi\int_0^1 (-x^2 + x)^2\,dx && \text{Substitute for } f(x). \\
&= \pi\int_0^1 (x^4 - 2x^3 + x^2)\,dx && \text{Expand integrand.} \\
&= \pi\left[\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3}\right]_0^1 && \text{Find antiderivative.} \\
&= \frac{\pi}{30} && \text{Apply Fundamental Theorem.} \\
&\approx 0.105 && \text{Round to three decimal places.}
\end{aligned}$$

So, the volume of the solid is about 0.105 cubic unit.



(a) Plane region



(b) Solid of revolution

FIGURE 5.27

Examples:

A regulation-size football can be modeled as a solid of revolution formed by revolving the graph of

$$f(x) = -0.0944x^2 + 3.4, \quad -5.5 \le x \le 5.5$$

about the x-axis, as shown in Figure 5.30. Use this model to find the volume of a football. (In the model, x and y are measured in inches.)

SOLUTION To find the volume of the solid of revolution, use the Disk Method.

$$\text{Volume} = \pi \int_{-5.5}^{5.5} [f(x)]^2 \, dx \qquad \text{Disk Method}$$

$$= \pi \int_{-5.5}^{5.5} (-0.0944x^2 + 3.4)^2 \, dx \qquad \text{Substitute for } f(x).$$

$$\approx 232 \text{ cubic inches} \qquad \text{Volume}$$

So, the volume of the football is about 232 cubic inches.

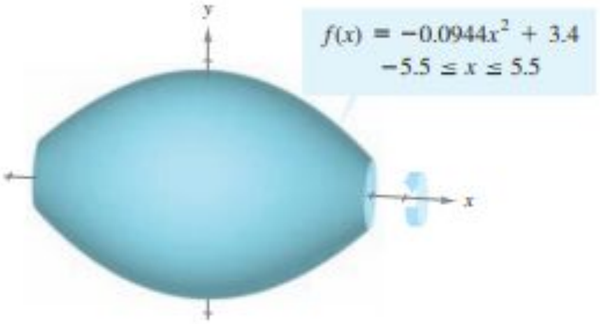


FIGURE 5.30 A football-shaped solid is formed by revolving a parabolic segment about the x-axis.

Practice:

1) Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x-axis about the x-axis.

2) Find the volume of the solid of revolution generated by rotating the curve $y = x^3$ between $y = 0$ and $y = 4$ about the y-axis.

Answer Key:

Once you have completed the problems, check your answers here.

1)

$$V = \int_a^b A(x)\,dx$$

$$= \pi \int_1^4 x^4 - 8x^3 + 26x^2 - 40x + 25\,dx$$

$$= \pi \left(\frac{1}{5}x^5 - 2x^4 + \frac{26}{3}x^3 - 20x^2 + 25x \right) \Big|_1^4$$

$$= \frac{78\pi}{5}$$

2)

$$\text{Vol} = \pi \int_c^d x^2\,dy$$

$$= \pi \int_0^4 y^{2/3}\,dy$$

$$= \pi \left[\frac{3y^{5/3}}{5} \right]_0^4$$

$$= \frac{3\pi}{5} \left[y^{5/3} \right]_0^4$$

$$= \frac{3\pi}{5} [10.079 - 0]$$

$$= 19.0 \text{ units}^3$$

Additional Practice:

In your Calculus book read through Section 7.2 and complete problems 1, 3, 7, 9, 11, and 13

Interactive Practice

Extra Practice with Answers